Exam. Code : 103204
Subject Code : 9034

B.A./B.Sc. 4th Semester (Old Syllabus 2014)

MATHEMATICS

Paper—I

(Statics & Solid Geometry)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all selecting at least **TWO** questions each from Sections A and B. All questions carry equal marks.

SECTION—A

1. (a) Two forces $\vec{P}$ and $\vec{Q}$ acting at a point have a resultant $\vec{R}$. If the magnitude of $\vec{P}$ is doubled, the magnitude of $\vec{R}$ is doubled and if magnitude of $\vec{Q}$ is doubled and reversed in direction, even then magnitude of $\vec{R}$ is doubled. Show that $P : Q : R = \sqrt{6} : \sqrt{2} : \sqrt{5}$.

   (b) $\vec{P}$ and $\vec{Q}$ are two components of a given force $\vec{F}$ and its line of action divides the angle between them in the ratio 1 : 2. Prove that $Q(F + Q) = P^2$.

5,5

2. (a) State and prove $\lambda - \mu$ Theorem.

(b) Two like parallel forces $\vec{P}$ and $\vec{Q}$ act at two points of body. If $\vec{Q}$ be changed to $\frac{P^2}{Q}$, show that the line of action of the resultant is same as it would be if the forces are simply interchanged. 5,5

3. (a) Forces of magnitude P, 3P, 2P and 5P act along the sides AB, BC, CD and DA of square ABCD. Find the magnitude and direction of their resultant and prove that it meets AD produced at a point E such that AE : ED = 5 : 4.

(b) Show that two coplanar couples of equal and opposite moments are in equilibrium. 5,5

4. (a) One end of a uniform rod of weight W is attached to a hinge and the other end is supported by a string attached to the extremity of the rod. If the rod and the string are inclined at the same angle α to the horizontal, then show that reaction of the hinge is $\frac{W}{4}\sqrt{8+\text{cosec}^2\alpha}$.

(b) How high can a particle rest inside a rough hollow sphere of radius a if the coefficient of friction is μ ? 5,5

5. (a) Show that the centre of gravity of a solid right circular cone lies on its axis and divides the axis in the ratio 3 : 1 from the vertex.

(b) If a piece of wire is bent into the shape of an isosceles triangle whose sides are a, a and b, show that the distance of centre of gravity from the base is $\frac{a}{2}\sqrt{\frac{2a-b}{2a+b}}$. 5,5

SECTION—B

6. (a) The axis of a right circular cylinder of radius 2 has equations $\frac{x-1}{2}=\frac{y}{3}=\frac{z-3}{1}$. Find its equation.

(b) Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y = 1$ having its generators parallel to the line $x = y = z$. 5,5

7. (a) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and whose guiding curve is the ellipse $2x^2 + 3y^2 = 4$, $z = 1$.

(b) Find the equation of the right circular cylinder whose axis $x - 2 = z$, $y = 0$ and passes through the point (3, 0, 0). 5,5

8. (a) The section of a cone whose vertex is P and guiding curve the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $z = 0$ by the plane $x = 0$ is a rectangular hyperbola. Show that the locus of P is $\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1$.

(b) Find the equation of the cone with vertex at the origin and which passes through the curve given by $x^2 + y^2 + z^2 + x - 2y + 3z = 4$ and $x^2 + y^2 + z^2 + 2x - 3y + 4z = 5$. 5,5

9. (a) Find the equation of the right circular cone whose vertex is at the $(1, -2, -1)$, semi-vertical angle is $60°$ and the line $\frac{x-1}{3}=\frac{y+2}{-4}=\frac{z+1}{5}$ as its axis.

(b) Prove that the equation $7x^2 + 2y^2 + 2z^2 - 10zx + 10xy + 26x - 2y + 2z - 17 = 0$ represents a cone whose vertex is $(1, -2, 2)$. 5,5

10. (a) Find the equation of reciprocal cone of the given equation of the cone $2x^2 + 3y^2 + 4z^2 + 2yz + 4zx + 6xy = 0$.

(b) If $x=\frac{1}{2}$, $y = z$ represents one of a set of three mutually perpendiculars generators of the cone $11yz + 6zx - 14xy = 0$, find the equations of the other two. 5,5